Eq. (4), is satisfactory in describing the overall phenomenon of the liquid film generation.

Conclusions

A theoretical formula for estimating the SMD, based on Dombrowski and Johns' hypothesis[4] regarding the thickness of a planar disintegrating liquid sheet, was derived for a Y-jet atomizer, considering a conical surface for the disintegrating sheet. The derived theoretical equation for the conical spray SMD atomizer includes geometrical parameters as well as easy-to-measure liquid fuel and atomizing gas flow parameters; and it correlates satisfactorily with experimental data obtained with water.

The only formulation available to date for estimating the SMD of Y-jet atomizers was that given by Wigg's equation.[2] It does not perform well for water. The present formulation has been shown to work for water, not only for Y-jet atomizers, but also for pressure swirl atomizers.[12] For pressure swirl atomizers, using experimental results from other sources, the formulation has been shown to perform reasonably for other liquids, such as diesel fuel, heavy fuel oil, ethyl alcohol, and kerosene. The formulation developed here can be applied for $W \gg 1$, which was the case in all of the experimental data presented, for which $33 < W < 161$.

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Optimal Performance of Enthalpy Rocket

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Introduction

THE theoretical performance of an enthalpy rocket has recently been discussed in several published Notes.[1–3] The rocket uses the stored energy of a thermal capacitor to heat a nonreacting working fluid. Parker and Humble[1] derived the velocity increment of a rocket that only consists of the capacitor and propellant masses. The released energy during the solidification of a suitable material heats the propellant from an absolute zero temperature to the material's melting temperature. Different materials and working gases have been compared; the heavier propellants appeared to hold some promise for applications requiring sizable velocity increments. Gany[2] has shown that the performance of an enthalpy rocket with no payload or additional inerts, other than the capacitor itself, is merely a function of capacitor and propellant thermodynamic properties. King[3] suggests some means to improve engine performance; in particular, he points out the benefit of a higher initial temperature of the working fluid and suggests a more complete use of the capacitor mass by allowing its temperature to range from values above the melting point to temperatures below it. Performance is also improved by heating the propellant to a lower temperature, i.e., by expelling the gas with a lower exhaust velocity but in larger quantities for unit mass of capacitor. King also proposes the use of nitrogen in the early phase of the engine operation and then a subsequent shift to hydrogen.

This Note investigates the theoretical upper limits of the enthalpy rocket by applying the theory of optimal control.[4] The assumptions of perfect heat transfer and negligible heat of vaporization for an initially liquid propellant do not reduce performance and appear to be coherent with the aim of the work. However, the application of the control theory has been directed to analyze the influence of the relevant parameters more than to obtain the ideal engine performance. In particular, this Note addresses the selection criteria for the capacitor material and working fluid, the influence of the propellant temperature inside its tank, and the best choice for the total temperature of the heated propellant, which is not constant during the engine operation.

Previous analyses have assumed that the whole rocket mass is functional, i.e., consisting of capacitor and propellant, to obtain the highest velocity increment; under the same conditions, conventional propulsion provides infinite velocity. The presence of a payload and inert mass apart from that of the capacitor is accounted for in this work and comparisons are more favorable for the enthalpy rocket. The tank mass is related to the physical state of the working fluid, and therefore, to its initial temperature; this aspect has not been considered.

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Statement of the Problem

A nonreacting working fluid is heated from its initial temperature T_i inside the tank to a total temperature T^o at the nozzle inlet by using the thermal energy that had previously been stored in the capacitor. Throughout, masses are expressed as fractions of the spacecraft initial mass, and the normalized mass of the exhausted propellant m_p is assumed to be the independent variable. The instantaneous rocket mass $m = 1 - m_p$ is composed of residual propellant, capacitor mass m_c, and other inert masses (payload, tank, etc.) m_t.

When only the heat of fusion r is used and x is the solid fraction of the capacitor material, the differential equations that express the conservation of energy and the spacecraft velocity V are

$$\frac{dx}{dm_p} = \frac{c_p(T^o - T_i)}{rm_c} \tag{1}$$

$$\frac{dV}{dm_p} = \frac{c}{m} = \frac{c}{1 - m_p} \tag{2}$$

where the effective exhaust velocity $c = \sqrt{(2c_pT^o)}$ also depends on the constant pressure specific heat $c_p = 2\gamma R_0/[\mu(\gamma - 1)]$, which is a function of the specific heat ratio γ, the molecular mass μ, and the universal gas constant R_0. If the temperature T_c of a capacitor material with specific heat c_c is allowed to differ from the melting temperature T_c^*, the following change of variables is carried out:

$$c_c\, dT_c = r\, dx \tag{3}$$

($x \le 0$ and $x \ge 1$ correspond to completely liquid and solid materials, respectively).

The theory of optimal control is applied to find the control law, i.e., T^o as a function of m_p, which maximizes the rocket final velocity with the constraint $T_i \le T^o \le T_c$. The Hamiltonian function is defined as

$$H = \lambda_x \frac{c_p(T^o - T_i)}{rm_c} + \lambda_V \frac{\sqrt{2c_pT^o}}{1 - m_p} \tag{4}$$

If one assumes that $T^o < T_c$ during the whole operation, the Euler–Lagrange equations are $\dot{\lambda}_x = \dot{\lambda}_V = 0$. Maximum final velocity is sought and $\lambda_V = 1$ is readily deduced; the unknown constant λ_x is provided by the optimization procedure.

If the working gas is initially at zero absolute temperature, the only control in Eq. (4) is the total enthalpy of the heated propellant $h = c_pT^o$, whose optimal value is obtained by equating to zero the partial derivative of the Hamiltonian with respect to h. One obtains

$$h = c_pT^o = \frac{1}{2}\left[\frac{rm_c}{\lambda_x(1 - m_p)}\right]^2 \tag{5}$$

The optimal control law provided by Eq. (5) can be fulfilled by arbitrarily selecting the propellant, i.e., c_p. The engine performance is independent of the selected propellant, but the total temperature of the exhausting gas is directly proportional to its molecular mass (and heavy propellants may violate the temperature constraint). Figure 1 presents the optimal temperature T^o for different propellants when a beryllium capacitor is cooled from 3000 to 600 K in three phases: cooling of the liquid material, solidification, and cooling of the solid beryllium. A changeover to a lighter propellant is necessary as soon as Eq. (5) provides $T^o > T_c$. Figure 1 also shows that the engine should be operated with different propellants if a constant value for T^o is arbitrarily assumed[3] for a nonoptimal operation; the propellant molecular mass should be progressively reduced.

A greater amount of propellant per unit capacitor mass can be handled by increasing the initial temperature T_i. In Eq. (4), which is rewritten as

$$H = \lambda_x \frac{h - c_pT_i}{rm_c} + \frac{\sqrt{2h}}{1 - m_p} \tag{6}$$

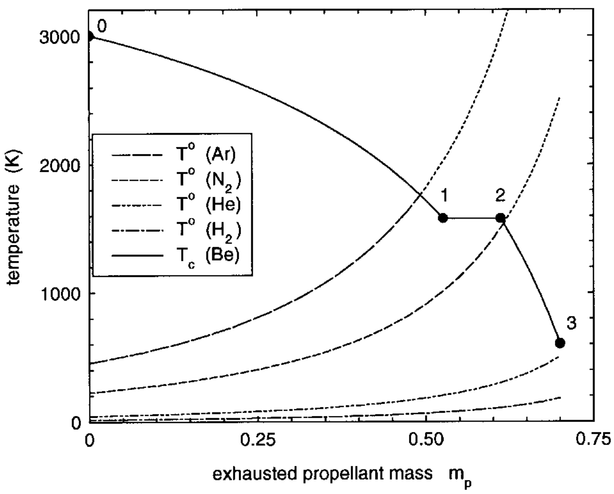

Fig. 1 Optimal control law for Be capacitor and different propellants (T_i = 0 K, $T_c \ge$ 600 K, m_t = 0.1, and m_c = 0.2).

c_p can be considered to be an additional control. Equation (6) is maximized by the highest c_p as $\lambda_x < 0$; according to Pontryagin's maximum principle, hydrogen is therefore the best propellant when $T_i > 0$.

The performance of a hydrogen-fed enthalpy rocket is improved if the capacitor thermal energy is further exploited by lengthening the third phase in Fig. 1 until $T^o = T_c$ and by adding a fourth phase with $T^o = T_c = T_c^* - (x - 1)r/c_c$. During this phase

$$\dot{\lambda}_x = -\frac{\partial H}{\partial x} = \lambda_x \frac{c_p/c_c}{m_c} + \frac{rc_p/c_c}{(1 - m_p)\sqrt{2c_pT^o}} \tag{7}$$

and λ_x is no longer constant. The procedure also provides the optimal capacitor mass; it is sufficient to consider an additional adjoint variable λ_m, whose value is provided by the differential equation

$$\dot{\lambda}_m = -\frac{\partial H}{\partial m_c} = \lambda_x \frac{\dot{x}}{m_c} \tag{8}$$

The differential problem requires four initial values for Eqs. (1) and (2) and Eqs. (7) and (8); the capacitor mass and the propellant masses that are exhausted in each phase are also unknown. These nine parameters are provided by the solution of a boundary-value problem; the nine boundary conditions are $V_0 = 0$, $x_0 = c_c(T_c^* - T_{c0})/r$, and $\lambda_{m0} = 0$ at the initial point; $x_1 = 0$ and $x_2 = 1$ at the beginning and the end of the solidification phase, respectively; $T_3^o = T_c^* - (x_3 - 1)r/c_c$ at the beginning of the fourth phase; and $\lambda_{x4} = 0$, $m_{p4} + m_c = 1 - m_t$, and $\lambda_{m4} + H_4 = 0$ at the final point.

In some cases, Eq. (5) may provide $T_0^o < T_i$, and the engine initially operates as a cold gas thruster until $T^o = T_i$ is found by means of Eq. (5). An example of this more complex case is presented in Fig. 2 for an assigned nonoptimal capacitor mass $m_c = 0.3$.

Results

The theoretical analysis has already shown that the lightest gas is the most appropriate propellant for an enthalpy rocket. High temperatures of the heated propellant produce high specific impulses I_{sp}, but low overall performance because of the heavy capacitor; low temperatures and large mass-flows are preferred, at least during early operation. An adequate temperature of the propellant entering the nozzle could be obtained by reducing the residence time inside the heat exchanger or by mixing the gas that leaves the capacitor with a suitable amount of cold gas that bypasses it. The bypass mass-flow is progressively reduced to increase the specific impulse until the bypass valve is completely closed ($T^o = T_c$); a final operation, with decreasing propellant and capacitor temperature, allows a further velocity increment.

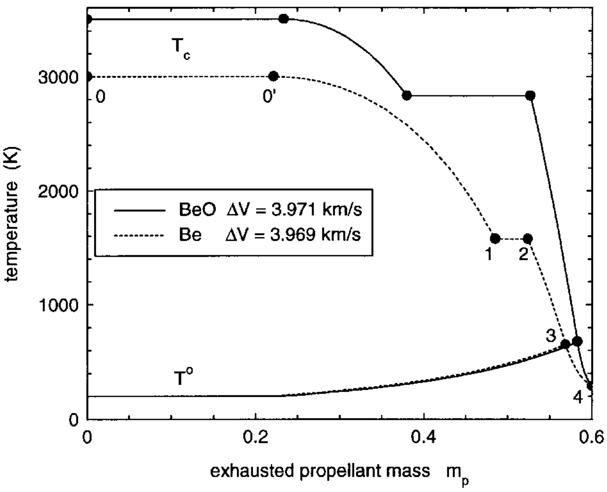

Fig. 2 Optimal control law for H_2 with different capacitor materials (T_i = 200 K, m_t = 0.1, and m_c = 0.3).

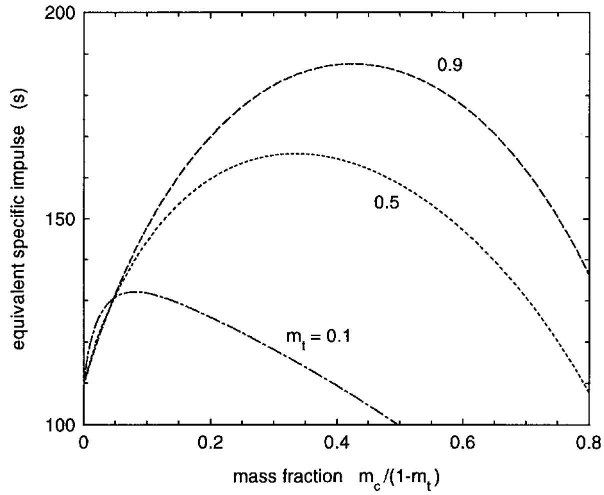

Fig. 4 Equivalent specific impulse using H_2 with Be capacitor (T_i = 20 K).

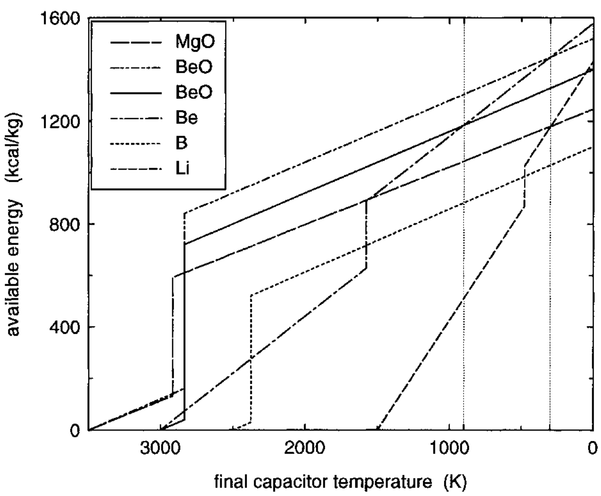

Fig. 3 Available energy using different capacitor materials.

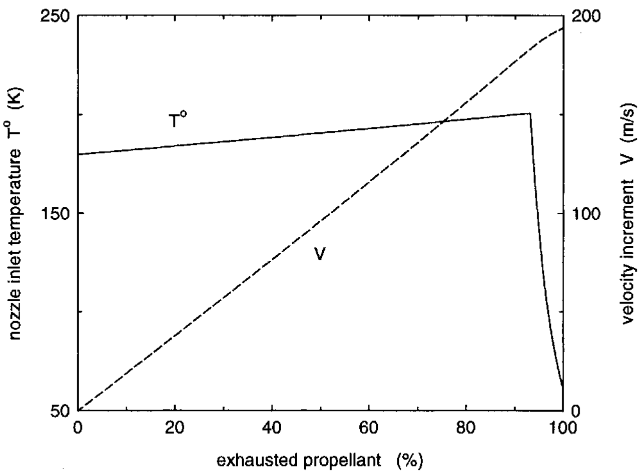

Fig. 5 Engine operation using H_2 with optimal Be capacitor mass (T_i = 20 K, m_t = 0.9, and m_c = 0.0427).

As far as the selection of the capacitor material is concerned, Fig. 3 shows the energy that is available from the capacitor as a function of the final temperature. The materials and their relevant properties have been derived from Ref. 3; the peak operating temperatures have been selected according to the same reference, but a lower temperature (3000 K) has also been considered for BeO. The available energy (and the available temperature gradient inside the exchanger, which is neglected here) constitutes the only useful criterion if $T^o < T_c$ during the whole engine operation, and the Berillium is superior to its oxide only if the final temperature is lower than 300 and 900 K for a BeO peak temperature of 3500 and 3000 K, respectively. The presence of the final phase ($T^o = T_c$) further favors BeO, which has lower specific heat, as the last portion of the stored energy be delivered at a higher T^o corresponding to a larger I_{sp} (see Fig. 2, where $T_4 \approx 288$ K).

To judge the capabilities of this engine concept, the rocket performance is better expressed as $c_{eq} = -\Delta V / \ln m_t$, i.e., by means of the equivalent I_{sp} or the specific impulse that produces the same velocity increment in a conventionally propelled rocket that has the same inert mass except the capacitor, i.e., the same m_t. The enthalpy rocket performance is improved by higher T_i but, unfortunately, less rapidly than the performance of a cold gas thruster. Figure 4 highlights the enthalpy rocket performance for $T_i = 20$ K and different m_t as a function of the mass fraction $m_c/(1 - m_t)$, which is reserved for the Be capacitor. The best performance, in comparison to a conventional engine, corresponds to the highest m_t, and the suggested application for an enthalpy rocket is therefore that of stationkeeping or orbit control rather than primary propulsion. Even though the equivalent I_{sp} is scarce, the presence of a maximum value for $m_c > 0$ points out that, in these conditions, the enthalpy rocket is superior to a cold-gas thruster with the same initial propellant temperature. If T_i is sufficiently increased, the optimal solution presents an initial phase with $T^o = T_i$ to lighten the rocket without consuming stored energy; when $T_i \approx 110$ K ($m_t = 0.9$) the optimal m_c becomes zero, and at higher temperatures, the cold-gas thruster is superior. The operation with $m_t = 0.9$ and optimal capacitor mass is shown in Fig. 5. One should note that the gas temperatures at the nozzle inlet are quite low; other useful capacitors or energy sources could be found on the spacecraft.

Conclusions

This analysis has shown that even the optimal performance of an ideal enthalpy rocket barely compares to the capabilities of a cold-gas thruster. An enthalpy rocket seems to be useless for primary propulsion; its application for stationkeeping or attitude control is more promising, in particular when the highest-performing propellant, that is, hydrogen, is stored in a liquid state. This kind of mission is characterized by a discontinuous use of the engine, and one can restore the thermal energy in the capacitor, which might become a storage device for a type of resistojet that does not rely on a dedicated powerplant. The duty cycles of the electrical generator and thruster are made independent by using the available power, when it is not needed by the payload, to partially liquefy the capacitor material that is maintained at melting temperature. The thermal energy is directly used to heat the propellant to the same temperature when the engine operation is required. High specific impulses are achievable; larger thrust levels than in a conventional resistojet could also be obtained if the stored energy can be rapidly delivered to the working fluid. This enthalpy rocket concept is suitable for multiburn orbit transfer; energy is collected and stored during a whole orbit and is released with a high specific impulse when the thrust can be efficiently used, e.g., near the perigee. A new optimization should be carried out for this different engine concept to find the capacitor mass that provides a satisfactory matching between the powerplant and thruster duty cycle.

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